

Recent Developments in the Measurement of Intellectual Influence

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Two Types of Ranking Problems

1. “Journal Ranking” Problems
2. “CV” or “Citation Records” Ranking Problems

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1. “Journal Ranking” Problems
 - . Palacios-Huerta and Volij (2004)
 - . Demange (2012)
2. “CV” or “Citation Records” Ranking Problems

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- . Demange (2012)

2. “CV” or “Citation Record” Ranking Problems

- . Chambers and Miller (2013)

- . Perry and Reny (2013)

1. Journal Ranking Problems

ISI Web of KnowledgeSM

Journal Citation Reports[®]

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2012 JCR Social Science Edition

Cited Journal: **AMERICAN ECONOMIC REVIEW**

Number of times articles published in 2012 (in journals below) cited articles published in AM ECON REV (in years below). ([How to read this table](#))

Journals 1 - 20 (of 1327) [1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10] Page 1 of 67

Impact	Citing Journal	Cited Year											
		All Yrs	2012	2011	2010	2009	2008	2007	2006	2005	2004	2003	Rest
	All Journals	27876	237	561	701	920	1032	1017	1053	1098	1243	1096	18918
2.792	AM ECON REV	738	123	38	31	45	60	52	36	24	25	26	278
	ALL OTHERS (677)	677	0	10	13	15	25	16	19	28	31	30	490
1.065	J ECON BEHAV ORGAN	521	2	13	20	22	28	28	25	32	29	22	300
0.509	ECON LETT	430	2	13	11	14	28	18	17	19	18	15	275
0.437	APPL ECON	362	0	1	0	5	7	10	8	7	12	9	303
1.331	EUR ECON REV	303	2	7	15	10	14	13	15	16	21	11	179
0.557	ECON MODEL	294	0	2	5	6	11	5	6	9	9	7	234
2.086	J INT ECON	252	2	5	17	10	27	0	6	12	22	21	122

- An example of a journal ranking problem:

	<i>JPE</i>	<i>AER</i>	<i>QJE</i>
<i>JPE</i>	100	100	150
<i>AER</i>	50	200	100
<i>QJE</i>	100	50	150

of citations **by** AER **to** JPE

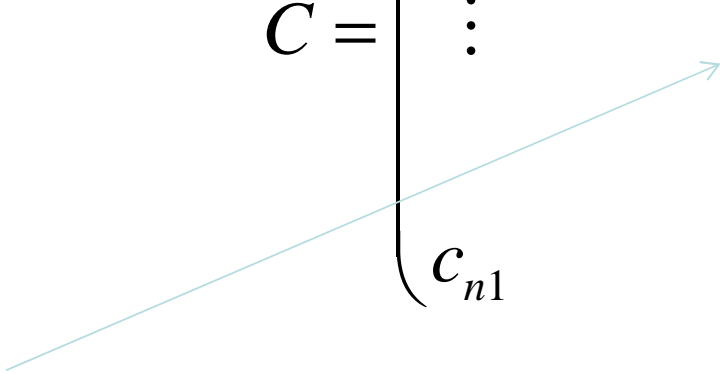
	<i>JPE</i>	<i>AER</i>	<i>QJE</i>
<i>JPE</i>	100	100	150
<i>AER</i>	50	200	100
<i>QJE</i>	100	50	150

This is the “**cv**” of the *JPE*. All its papers have been collapsed into one row. It shows the number of citations obtained from other journals.

	<i>JPE</i>	<i>AER</i>	<i>QJE</i>
<i>JPE</i>	100	100	150
<i>AER</i>	50	200	100
<i>QJE</i>	100	50	150

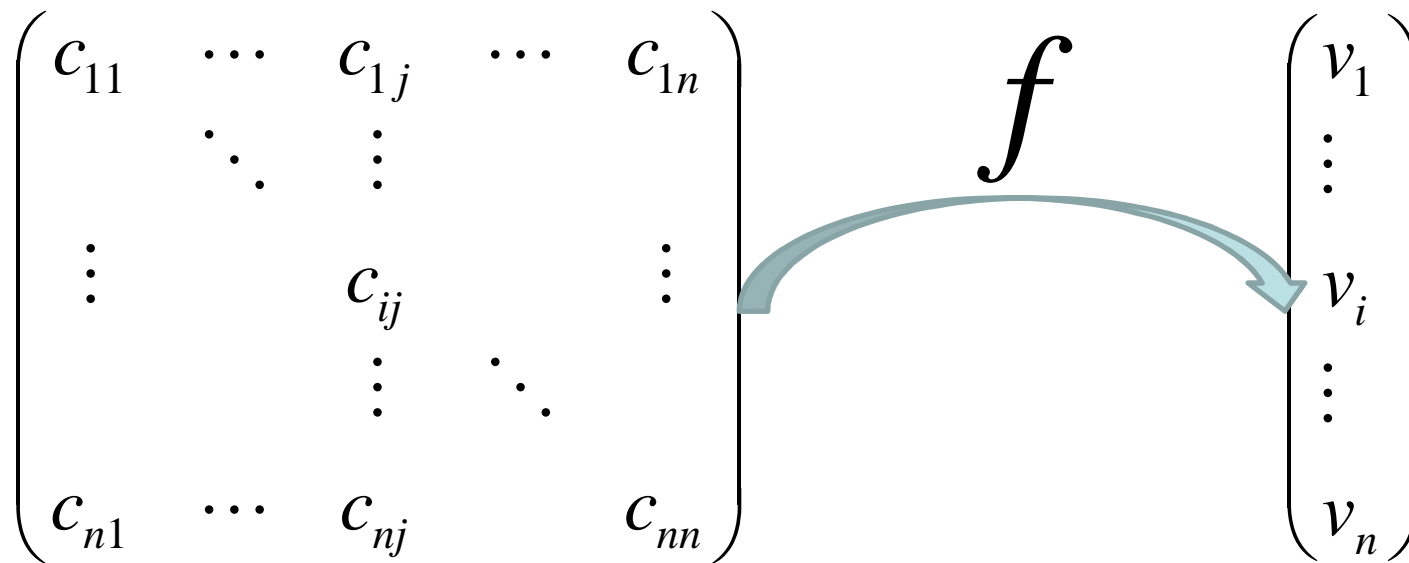
These are the *QJE*'s “**opinions**” about the journals.

- A journal ranking problem consists of a set of journals and their opinions about them.

$$C = \begin{pmatrix} c_{11} & \dots & c_{1n} \\ \vdots & \ddots & \vdots \\ c_{n1} & \dots & c_{nn} \end{pmatrix}$$


of citations **by** journal j **to** journal i

- The objective is to take a journal ranking problem and aggregate the journals' opinions into one "objective" opinion.



$$v_i \geq 0 \quad \sum_i v_i = 1$$

Examples

Ex.1: The impact factor

$$\begin{pmatrix} c_{11} & \cdots & c_{1j} & \cdots & c_{1n} \\ & \ddots & \vdots & & \vdots \\ \vdots & & c_{ij} & & \vdots \\ & & \vdots & \ddots & \\ c_{n1} & \cdots & c_{nj} & & c_{nn} \end{pmatrix} \xrightarrow{\quad} \begin{pmatrix} \sum_j c_{1j} \\ \vdots \\ \sum_j c_{ij} \\ \vdots \\ \sum_j c_{nj} \end{pmatrix} / \sum_{ij} c_{ij}$$

Ex.2: Liebowitz-Palmer method

$$\begin{pmatrix} c_{11} & \cdots & c_{1j} & \cdots & c_{1n} \\ & \ddots & \vdots & & \\ \vdots & & c_{ij} & & \vdots \\ & & \vdots & \ddots & \\ c_{n1} & \cdots & c_{nj} & & c_{nn} \end{pmatrix} \xrightarrow{\quad} \begin{pmatrix} v_1 \\ \vdots \\ v_i \\ \vdots \\ v_n \end{pmatrix}$$

$$v_i = \lambda \sum_j c_{ij} v_j \quad i = 1, \dots, n$$

$$\sum_i v_i = 1$$

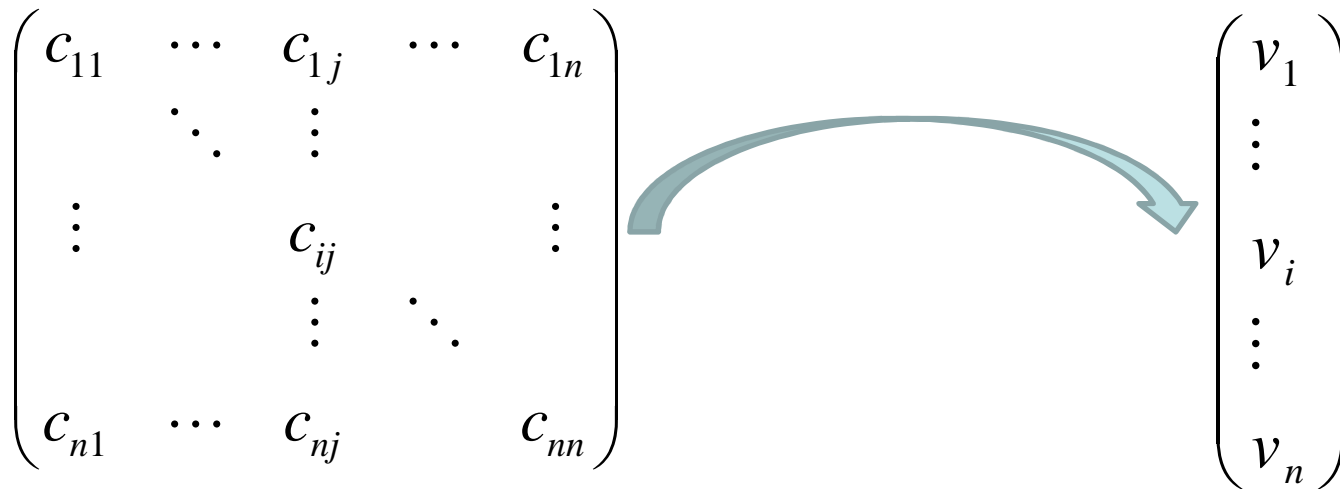
$$\begin{pmatrix} c_{11} & \cdots & c_{1j} & \cdots & c_{1n} \\ & \ddots & \vdots & & \\ \vdots & & c_{ij} & & \vdots \\ & & \vdots & \ddots & \\ c_{n1} & \cdots & c_{nj} & & c_{nn} \end{pmatrix}$$


 c_1
 c_j
 c_n

$$c_j = \sum_i c_{ij}$$

Ex.3: Invariant method

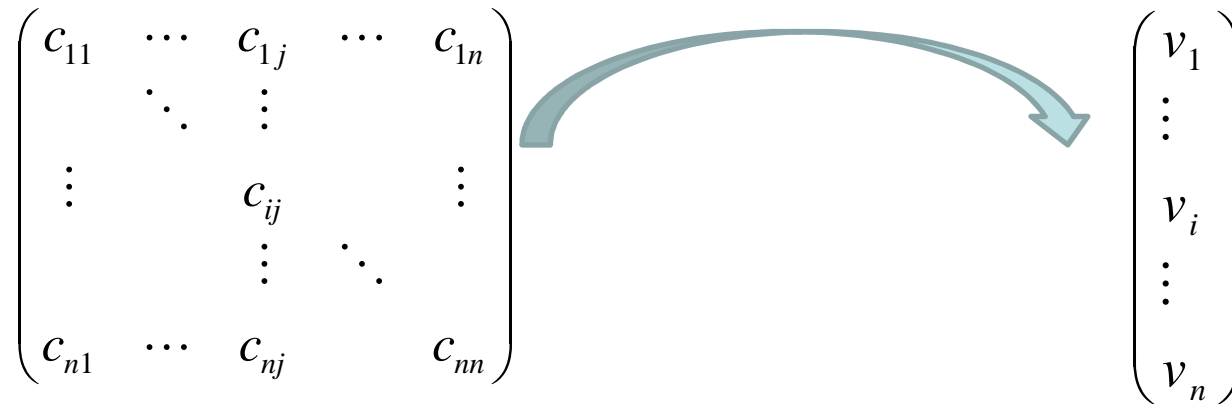
(Google's Page Rank, Eigenfactor)



$$v_i = \sum_j \frac{c_{ij}}{c_j} v_j \quad i = 1, \dots, n$$

$$\sum_i v_i = 1$$

Ex.4: The Handicap method



$$v_i = \sum_j \frac{c_{ij}}{q_j} \quad i = 1, \dots, n$$

$$q_j = \sum_i \frac{c_{ij}}{v_i} \quad j = 1, \dots, n$$

$$\sum_i v_i = 1$$

Special Problems

- **Normalized problems:** Ones that for each column, the sum of its entries equals 1.

$$\begin{array}{c} \left(\begin{array}{ccccc} c_{11} & \cdots & c_{1j} & \cdots & c_{1n} \\ & \ddots & \vdots & & \\ \vdots & & c_{ij} & & \vdots \\ & & \vdots & \ddots & \\ c_{n1} & \cdots & c_{nj} & & c_{nn} \end{array} \right) \\ \hline 1 \quad \cdots \quad 1 \quad \cdots \quad 1 \end{array}$$

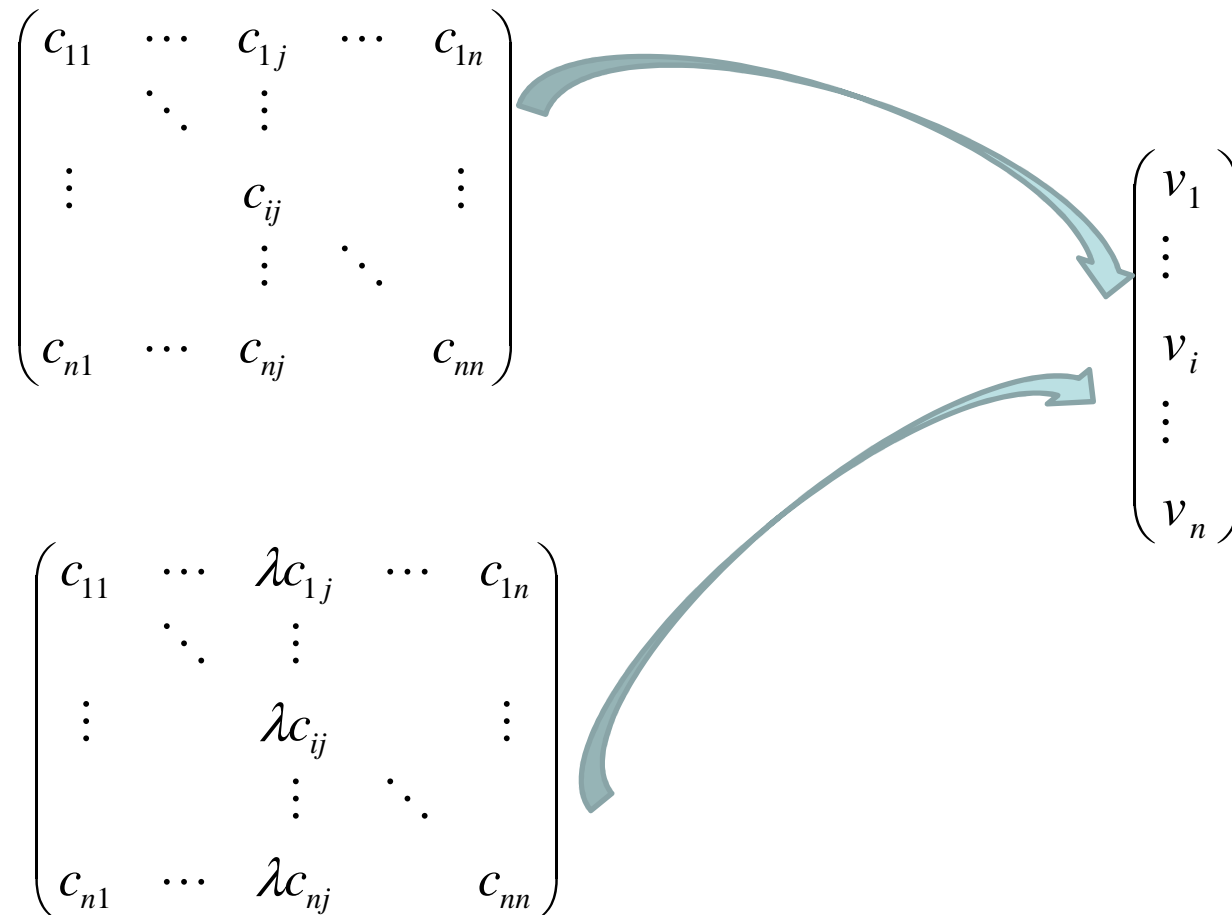
Special Problems

- **Row-Balanced problems:** Ones that for each row, the sum of its entries are all equal.

$$\begin{pmatrix} c_{11} & \cdots & c_{1j} & \cdots & c_{1n} \\ & \ddots & \vdots & & \\ \vdots & & c_{ij} & & \vdots \\ & & \vdots & \ddots & \\ c_{n1} & \cdots & c_{nj} & & c_{nn} \end{pmatrix} \begin{matrix} c \\ c \\ c \\ c \\ c \end{matrix}$$

Axioms

- **Invariance to reference intensity.** You have 1 vote and your opinion (something relative) does not change, so the scores should not change



Axioms

- **Uniformity**: If a problem is both normalized and row-balanced, then all journals have the same score.

If

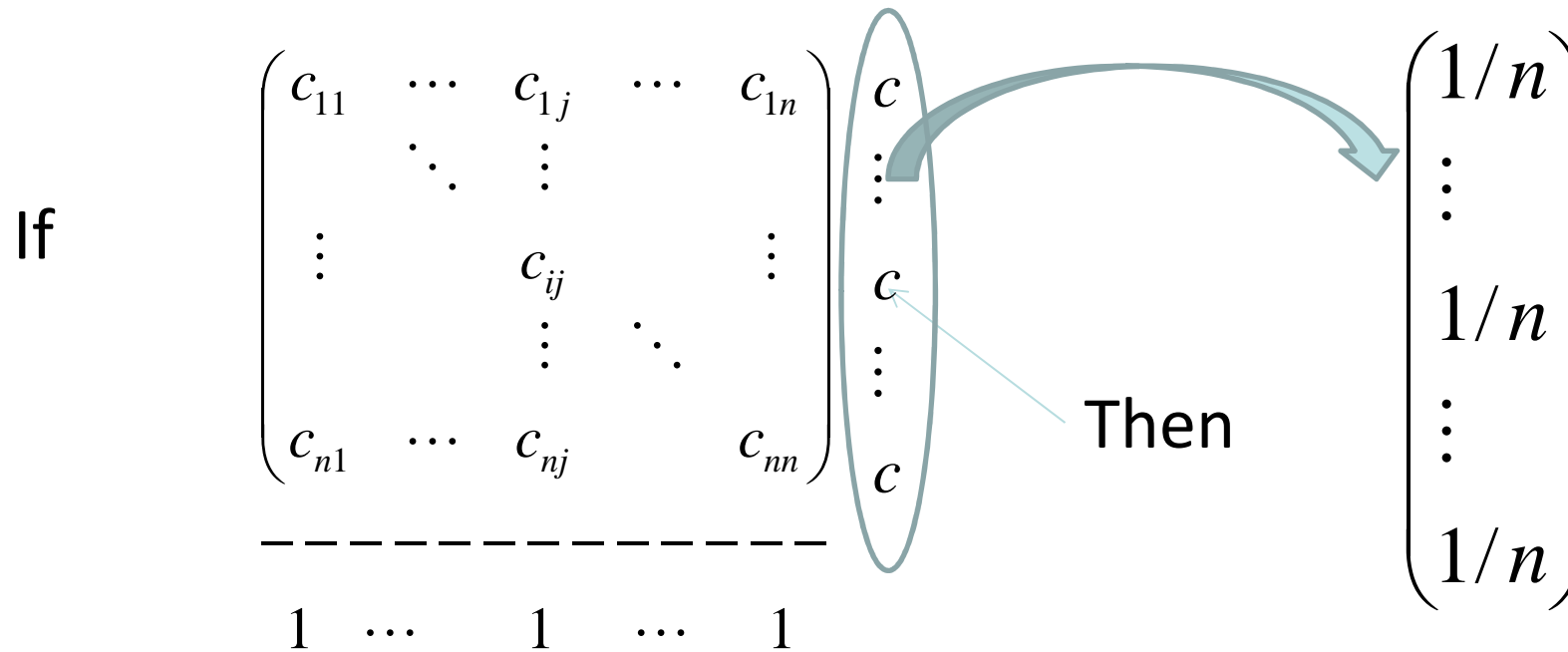
$$\begin{array}{cccccc}
 \left(\begin{array}{ccccc}
 c_{11} & \cdots & c_{1j} & \cdots & c_{1n} \\
 & \ddots & \vdots & & \\
 \vdots & & c_{ij} & & \vdots \\
 & & \vdots & \ddots & \\
 c_{n1} & \cdots & c_{nj} & & c_{nn}
 \end{array} \right) & \begin{array}{c} c \\ c \\ c \\ c \\ c \end{array} \\
 \hline
 & \begin{array}{ccccc}
 1 & \cdots & 1 & \cdots & 1
 \end{array}
 \end{array}$$

Then

$$\begin{array}{c}
 \left(\begin{array}{c}
 1/n \\
 \vdots \\
 1/n \\
 \vdots \\
 1/n
 \end{array} \right)
 \end{array}$$

Axioms

- **Exactness:** If a problem is normalized and if all journals have the same score, then the problem must be row-balanced.



Axioms

- **Homogeneity:** If everybody's opinion about a journal changes by a factor of λ , then his score in the aggregate also changes by a factor of λ .

If

$$\begin{pmatrix} c_{11} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots & & \vdots \\ c_{i1} & \cdots & c_{ij} & \cdots & c_{in} \\ \vdots & & \vdots & \ddots & \vdots \\ c_{n1} & \cdots & c_{nj} & & c_{nn} \end{pmatrix} \rightarrow \begin{pmatrix} v_1 \\ \vdots \\ v_i \\ \vdots \\ v_n \end{pmatrix}$$

then

$$\begin{pmatrix} c_{11} & \cdots & c_{1j} & \cdots & c_{1n} \\ \vdots & \ddots & \vdots & & \vdots \\ \lambda c_{i1} & \cdots & \lambda c_{ij} & \cdots & \lambda c_{in} \\ \vdots & & \vdots & \ddots & \vdots \\ c_{n1} & \cdots & c_{nj} & & c_{nn} \end{pmatrix} \rightarrow \begin{pmatrix} v_1 \\ \vdots \\ \lambda v_i \\ \vdots \\ v_n \end{pmatrix}$$

Axioms

- **Weak homogeneity** (or “Import-Export” axiom)


Stigler, Stigler, Friedland (1990): In 2-journal problems, when journals have the *same* length of references, the relative impact is determined by their “import-export” ratio:

$$\begin{pmatrix} 1-a & b \\ a & 1-b \end{pmatrix} \xrightarrow{\quad} \begin{pmatrix} b \\ a \end{pmatrix} / (a+b)$$

Journal 1 imports b citations from 2, and exports a citations to 1.
Journal 2 imports a citations from 1, and exports b citations to 2.

Axioms

Consistency

If $\begin{pmatrix} 30 & 16 & 10 \\ 15 & 35 & 20 \\ 15 & 9 & 30 \end{pmatrix}$  $\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

then

$$\begin{pmatrix} 30 + 5 & 16 + 3 \\ 15 + 10 & 35 + 6 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

Results

- The **Invariant method** is the only one that satisfies *invariance to reference intensity*, *weak homogeneity* and *consistency* (Palacios-Huerta and Volij, 2004).

It also satisfies *uniformity* and *exactness*.

- The **Handicap method** is the only one that satisfies *invariance to reference intensity*, *uniformity* and *homogeneity* (Demange 2012).
- The **Handicap method** is the only one that satisfies *invariance to reference intensity*, *exactness* and *homogeneity* (Demange 2012).

- The Invariant Method does not satisfy homogeneity.
- The Handicap Method does not satisfy weak homogeneity (“import-export”) and does not satisfy consistency.

2- Ranking CVs or citation records
or scholars

A **citation record**, a.k.a. **CV**, is a list

$$CV = (c_1, \dots, c_n)$$

of non-negative numbers, ordered from highest to lowest, where n is the number of publications and, for each $i=1, \dots, n$, c_i is the number of citations obtained by publication i .

Examples of three CVs with different number of publications:

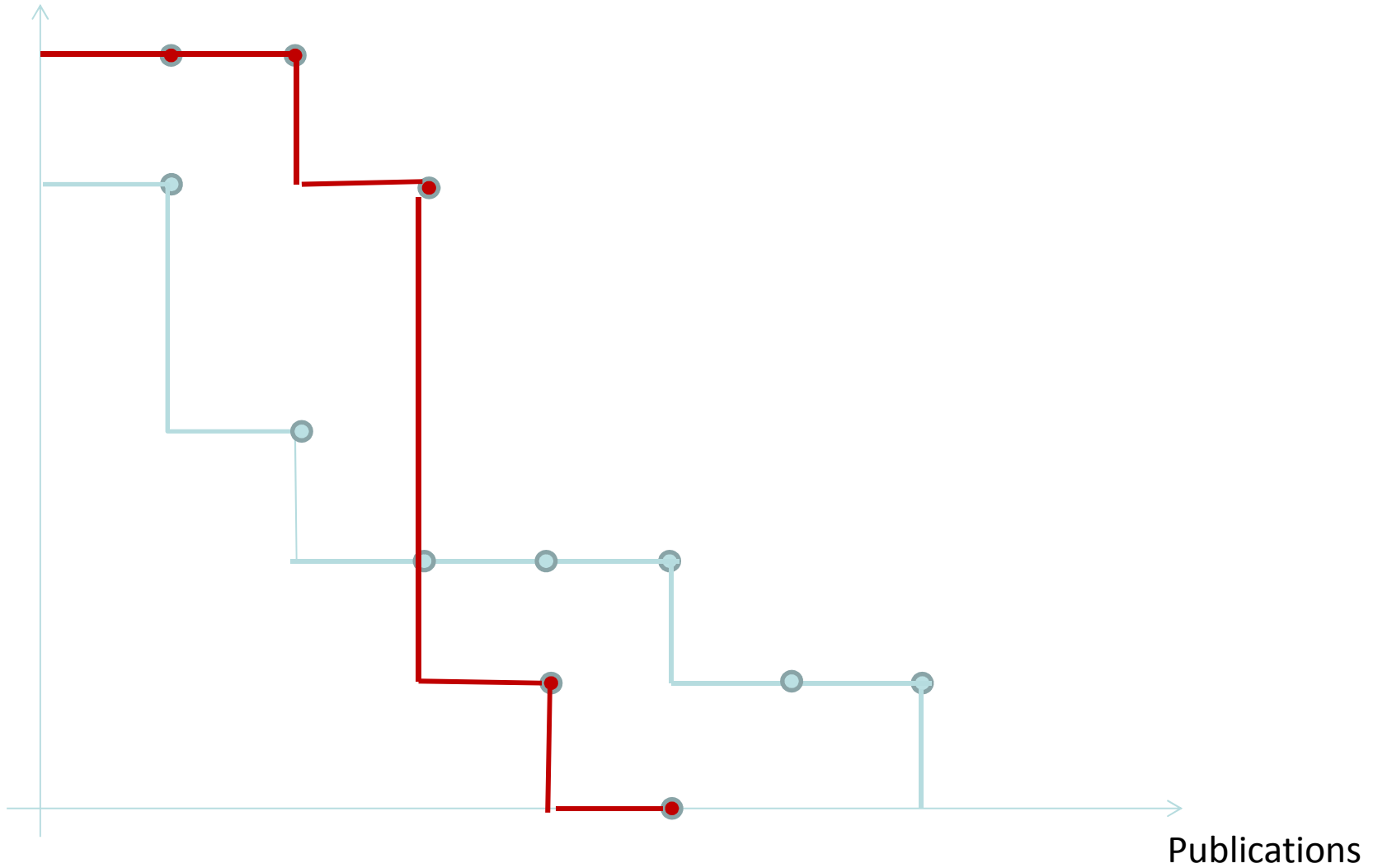
$$CV1 = (100, 2, 1, 0)$$

$$CV2 = (\sqrt{2}, 1, 1/3)$$

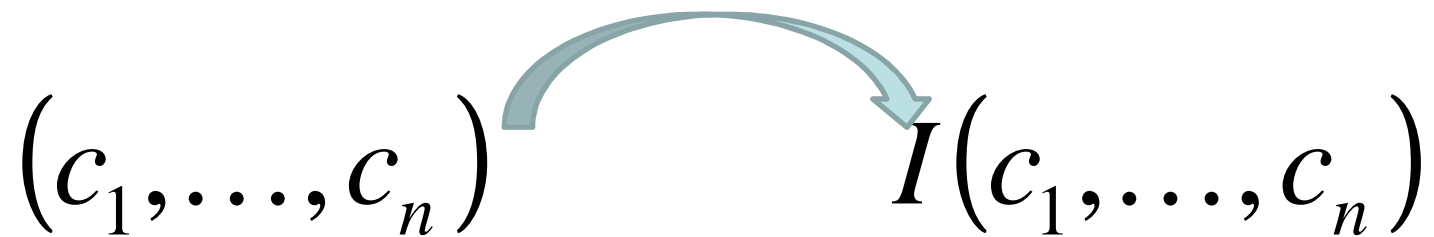
$$CV3 = (50, 49, 48, 30, 20)$$

Graphical representation of a CV

Citations



- The objective is to take a CV ranking problem and aggregate the CV's citations into one “objective” number.



Two families of Indices

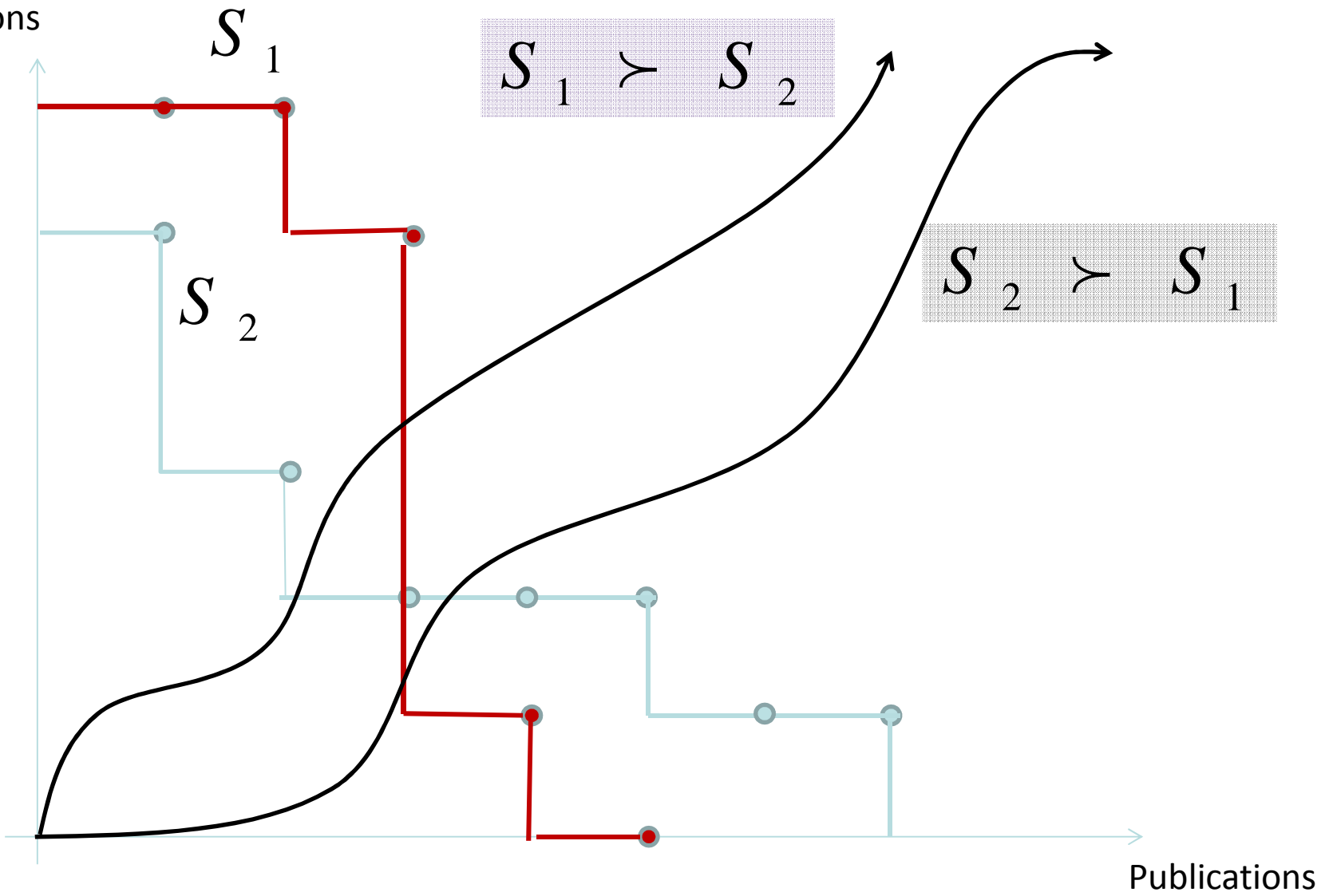
- A. Step-Based Indices
- B. Measure-Based Indices

A. Step-based Indices

(Chambers and Miller, 2013)

- Any **non-decreasing line** from the origin defines an index as follows: The index of a CV is the *length of the line* that is “inside” the CV’s frontier.

Citations

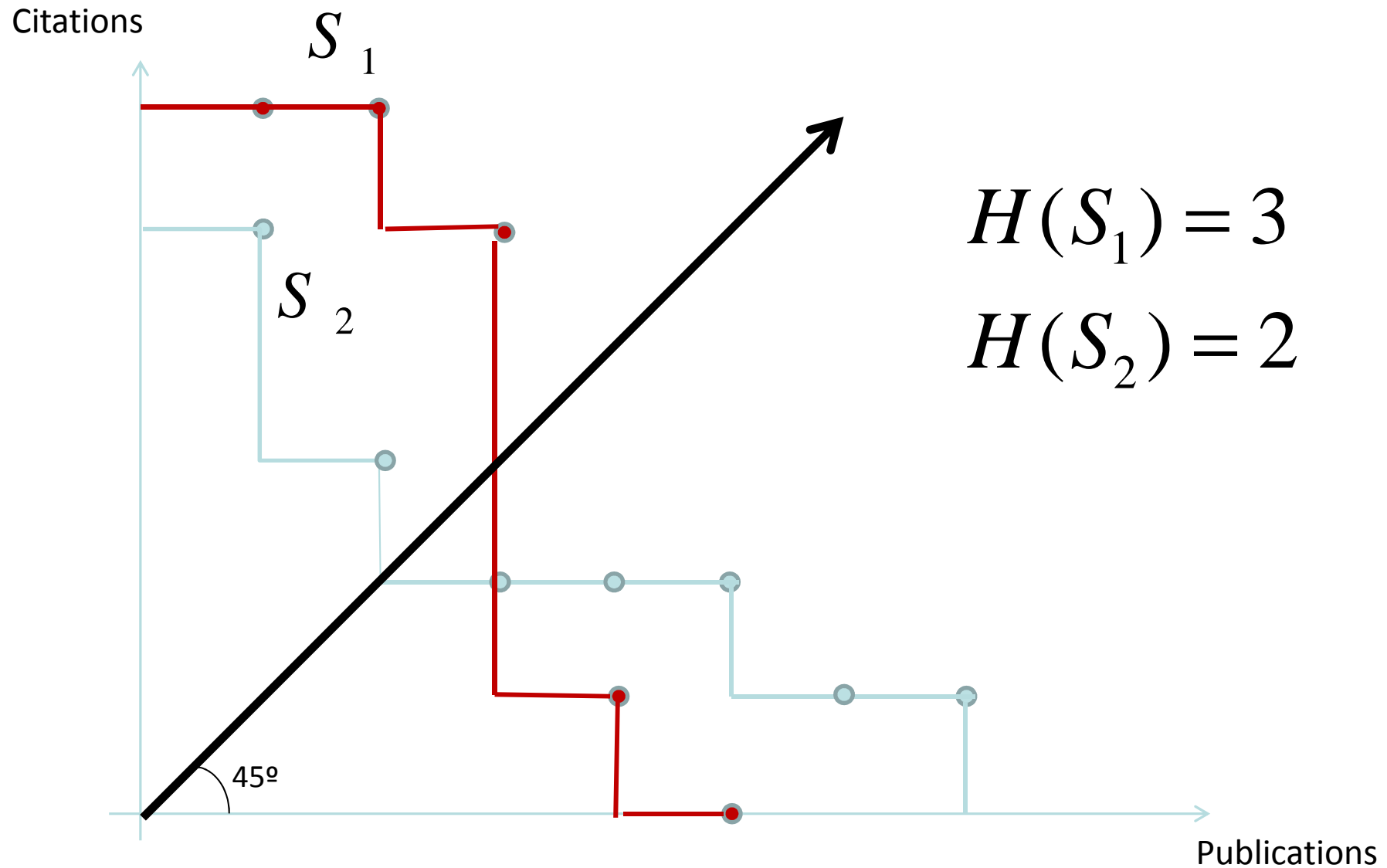


$S_1 \succ S_2$

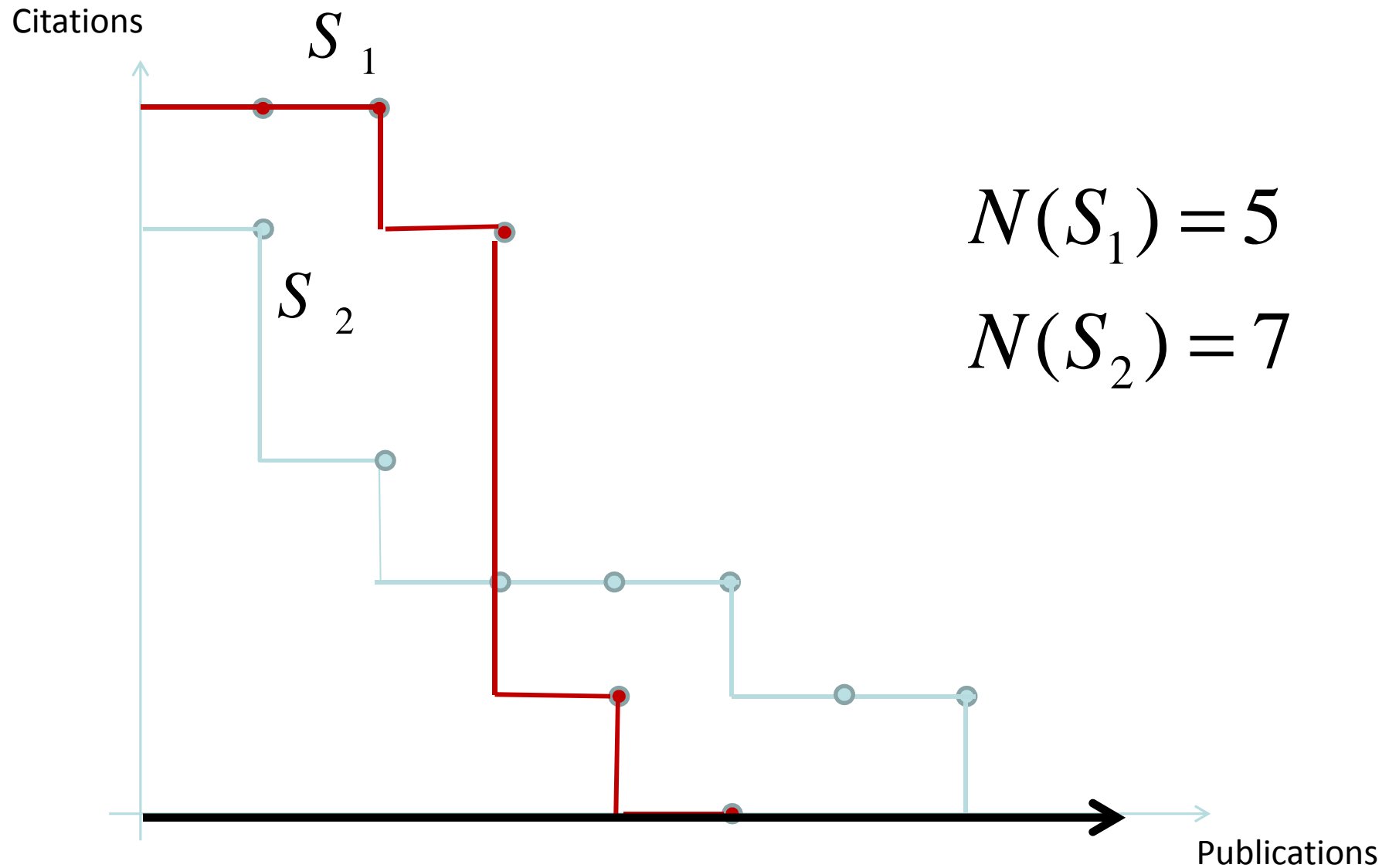
$S_2 \succ S_1$

Publications

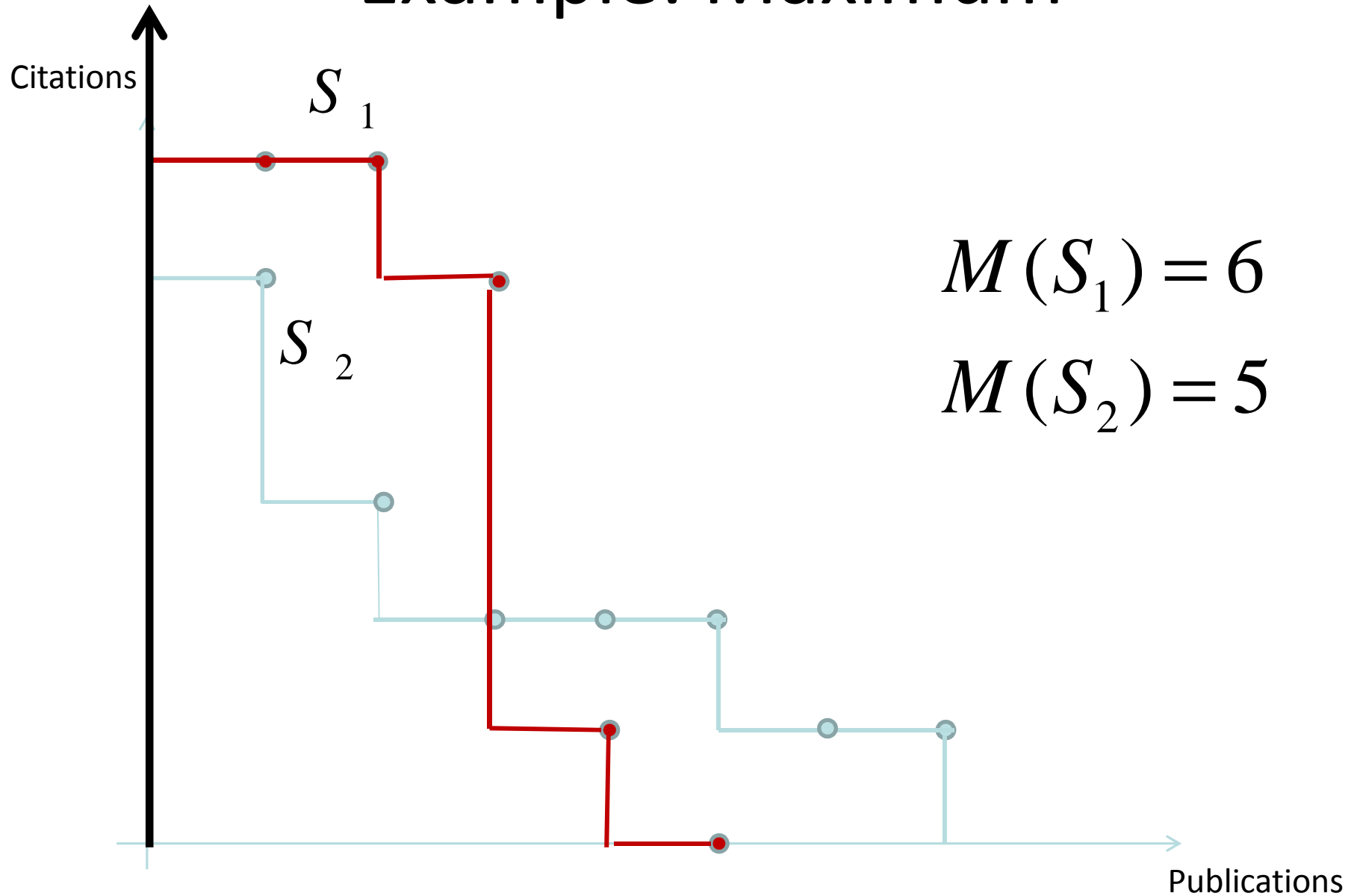
Example: H-index (Hirsch, 2005)



Example: Publication count

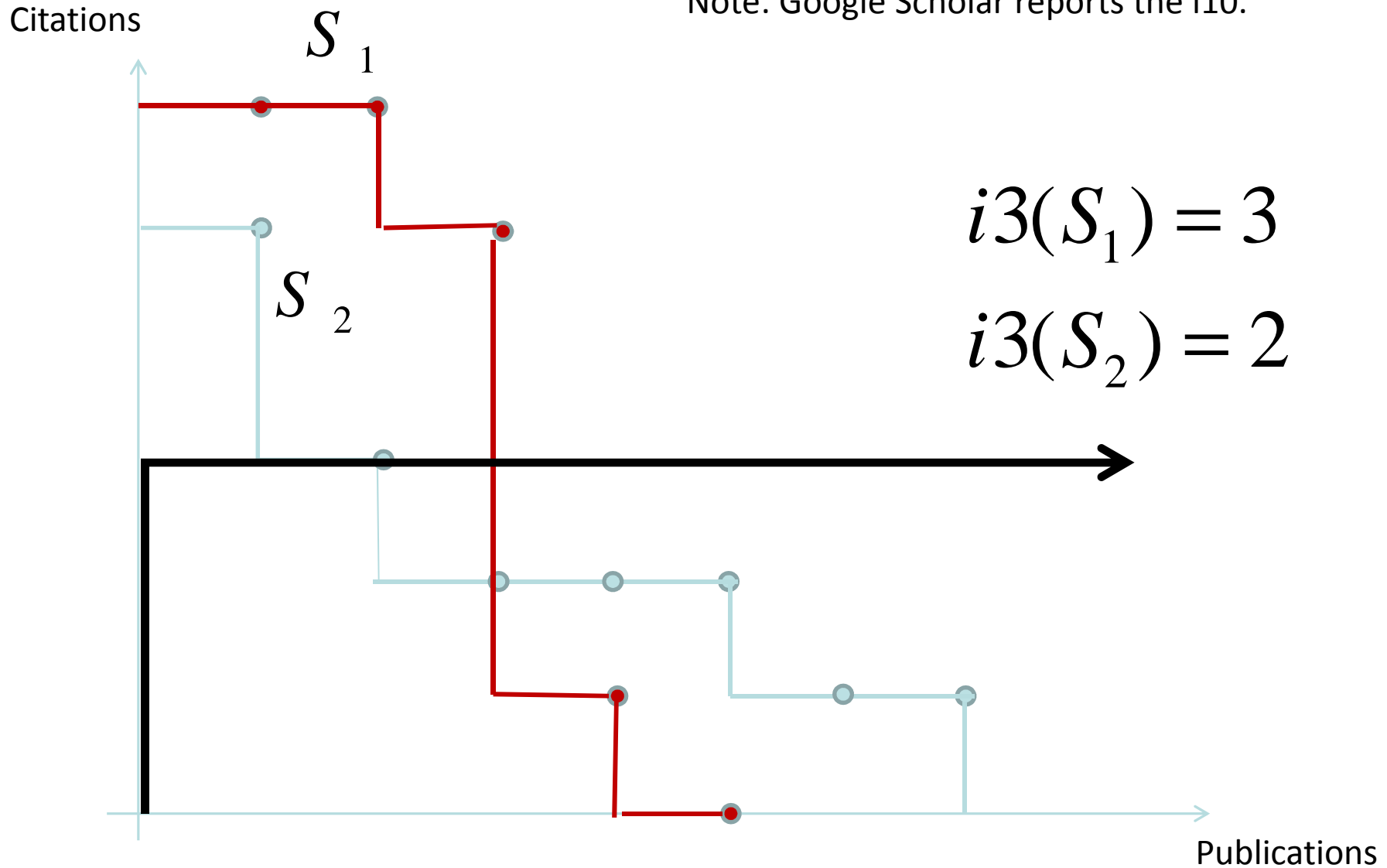


Example: Maximum



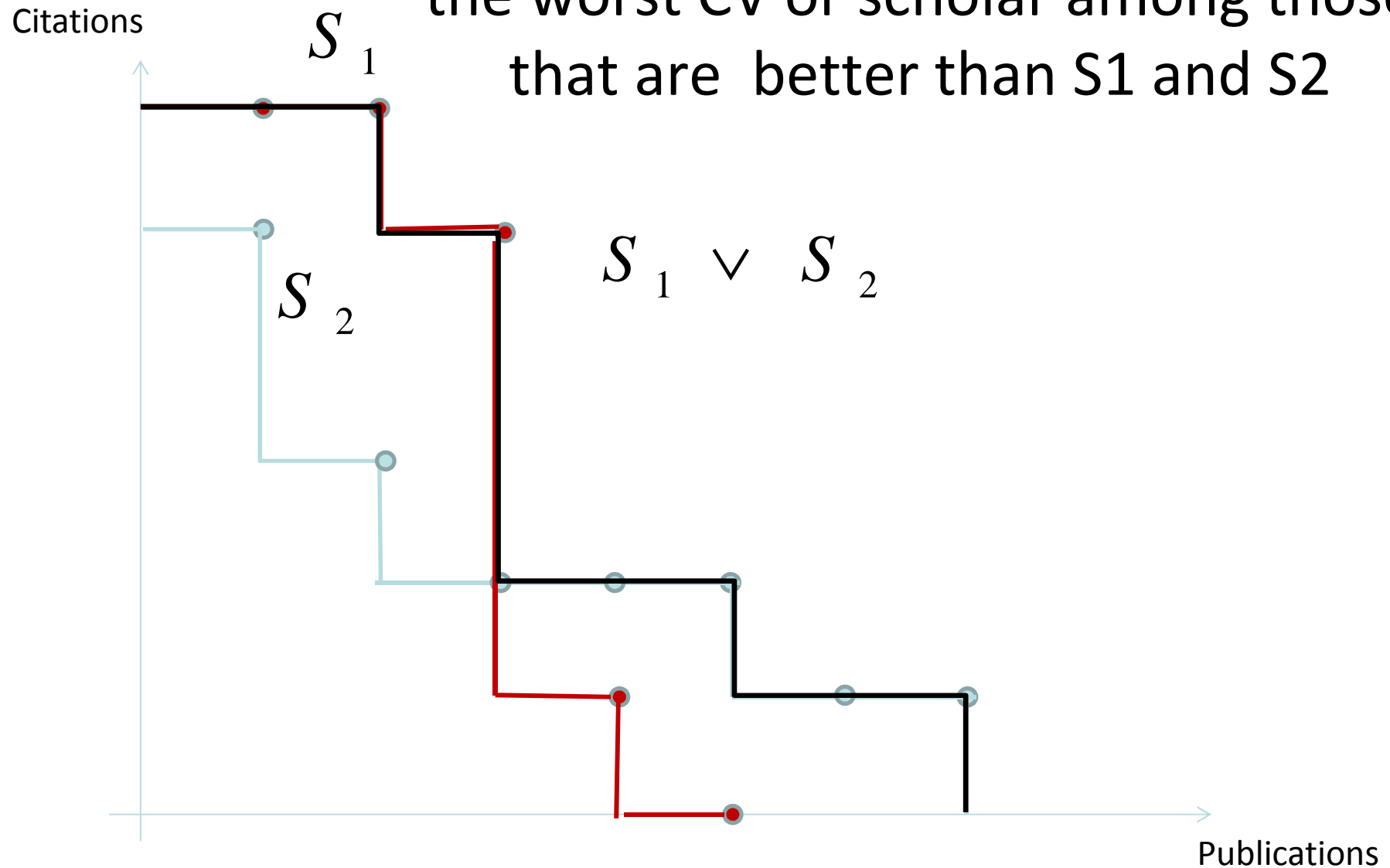
Example: i3 index (# of papers with at least 3 citations)

Note: Google Scholar reports the i10.



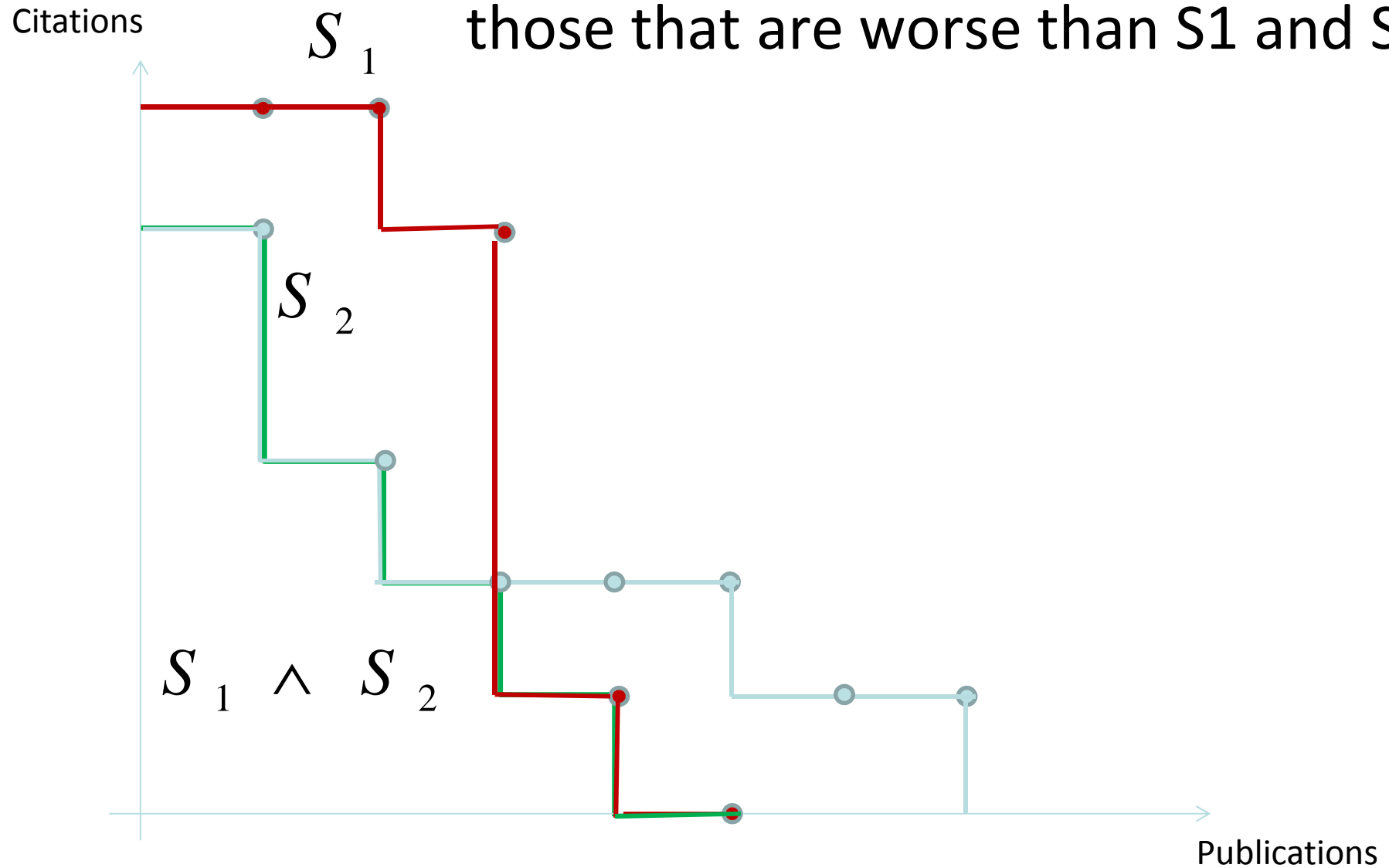
Now consider:

the worst CV or scholar among those that are better than S_1 and S_2



And:

the best CV or scholar among those that are worse than S_1 and S_2



Axioms

Consistency with better scholars and

Consistency with worse scholars

$$S_1 \succ S_2 \Rightarrow \begin{cases} S_1 \sim S_1 \vee S_2 \\ S_2 \sim S_1 \wedge S_2 \end{cases}$$

Result (Chambers-Miller, 2013)

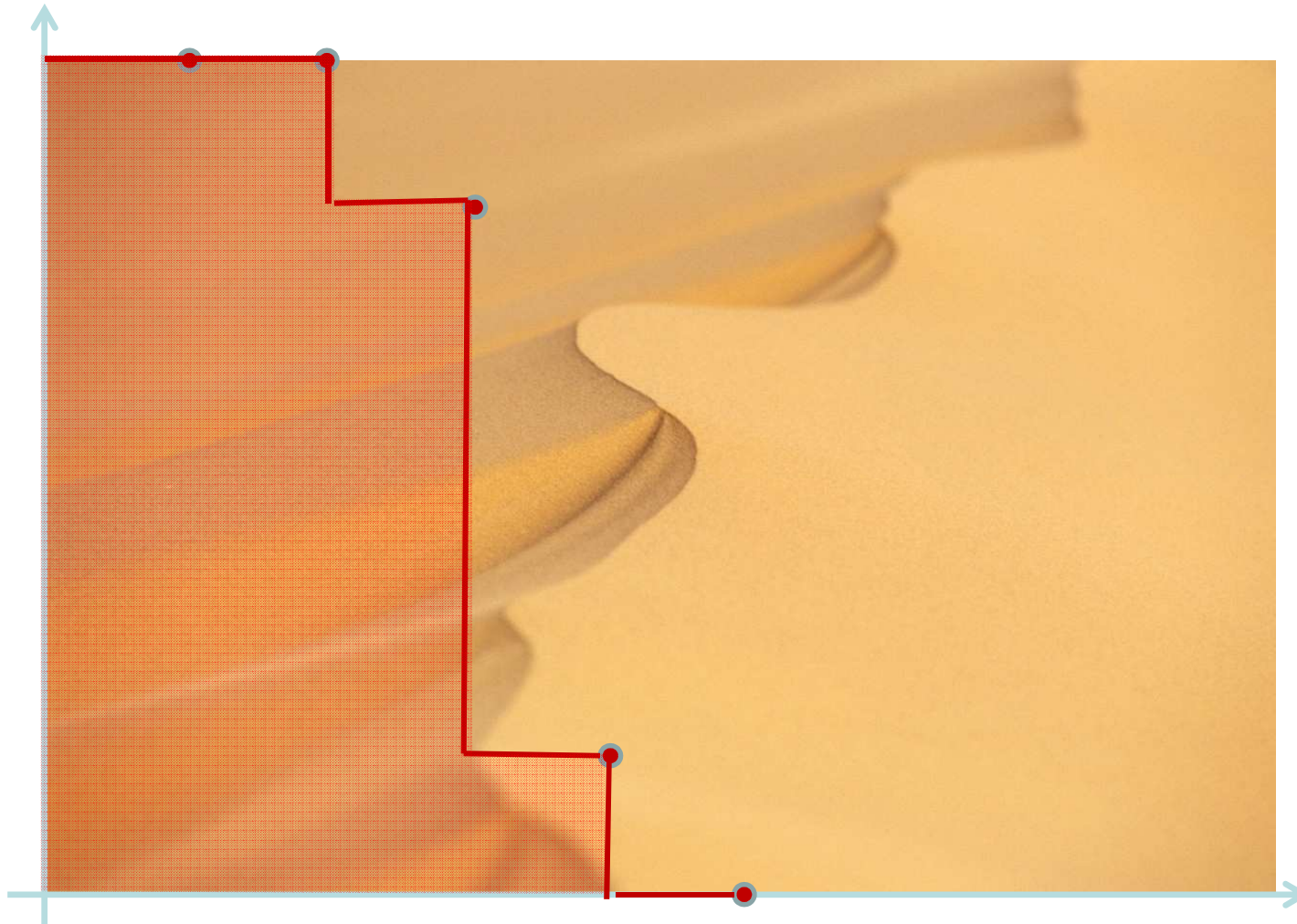
- A citation index satisfies **Consistency with better scholars** and **Consistency with worse scholars** (and a full range axiom) if and only if it is a *monotonic transformation* of a step-based index.

B. Measure-based indices

- Any **measure** on the positive orthant defines an **index** as follows: the index of the CV is the *measure of the space* under the CV's frontier.

Measure-based indices (“deserts”)

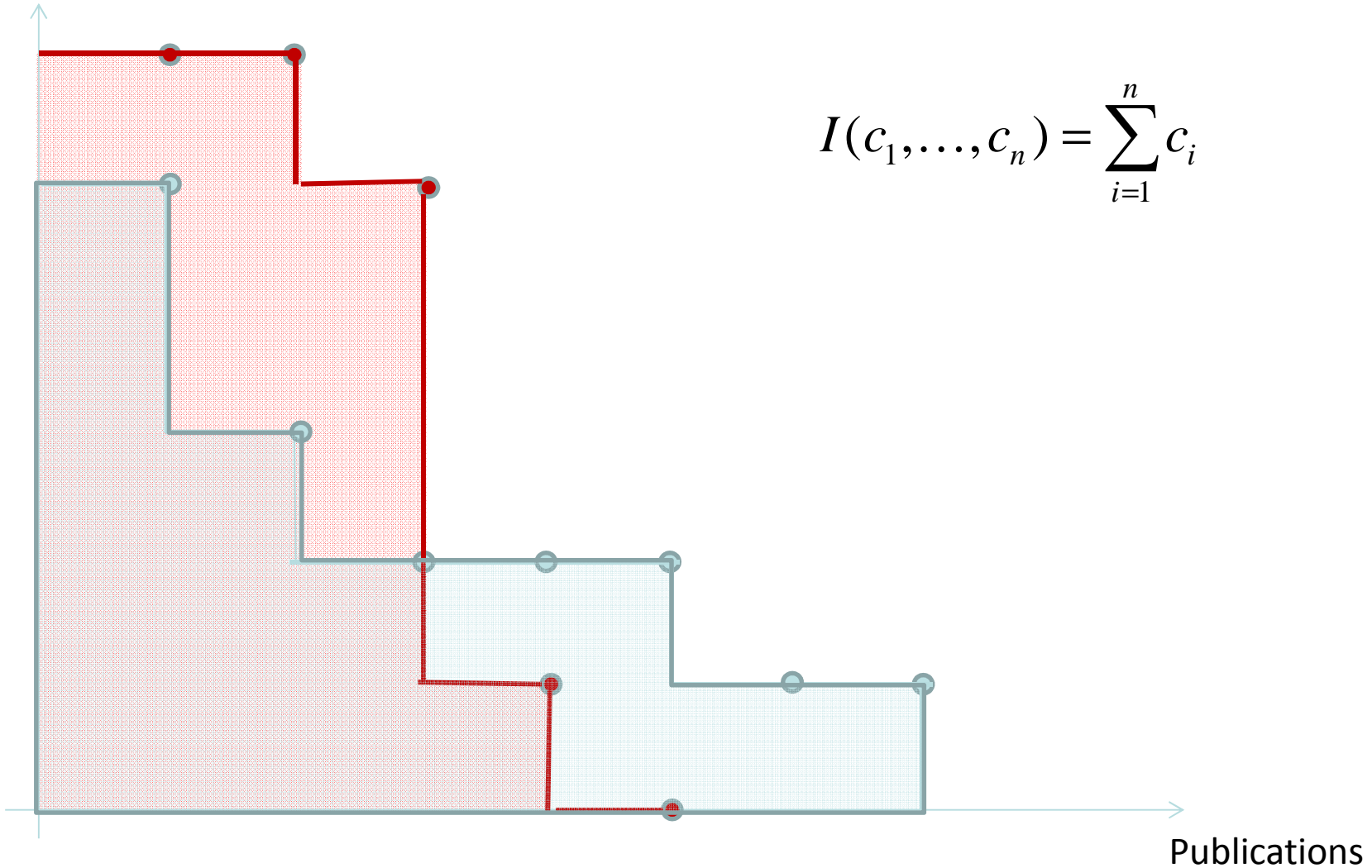
Citations



Publications

The Lebesgue measure (uniform) yields the citation count

Citations



$$I(c_1, \dots, c_n) = \sum_{i=1}^n c_i$$

Publications

Perry and Reny (2013) use the following product measure:

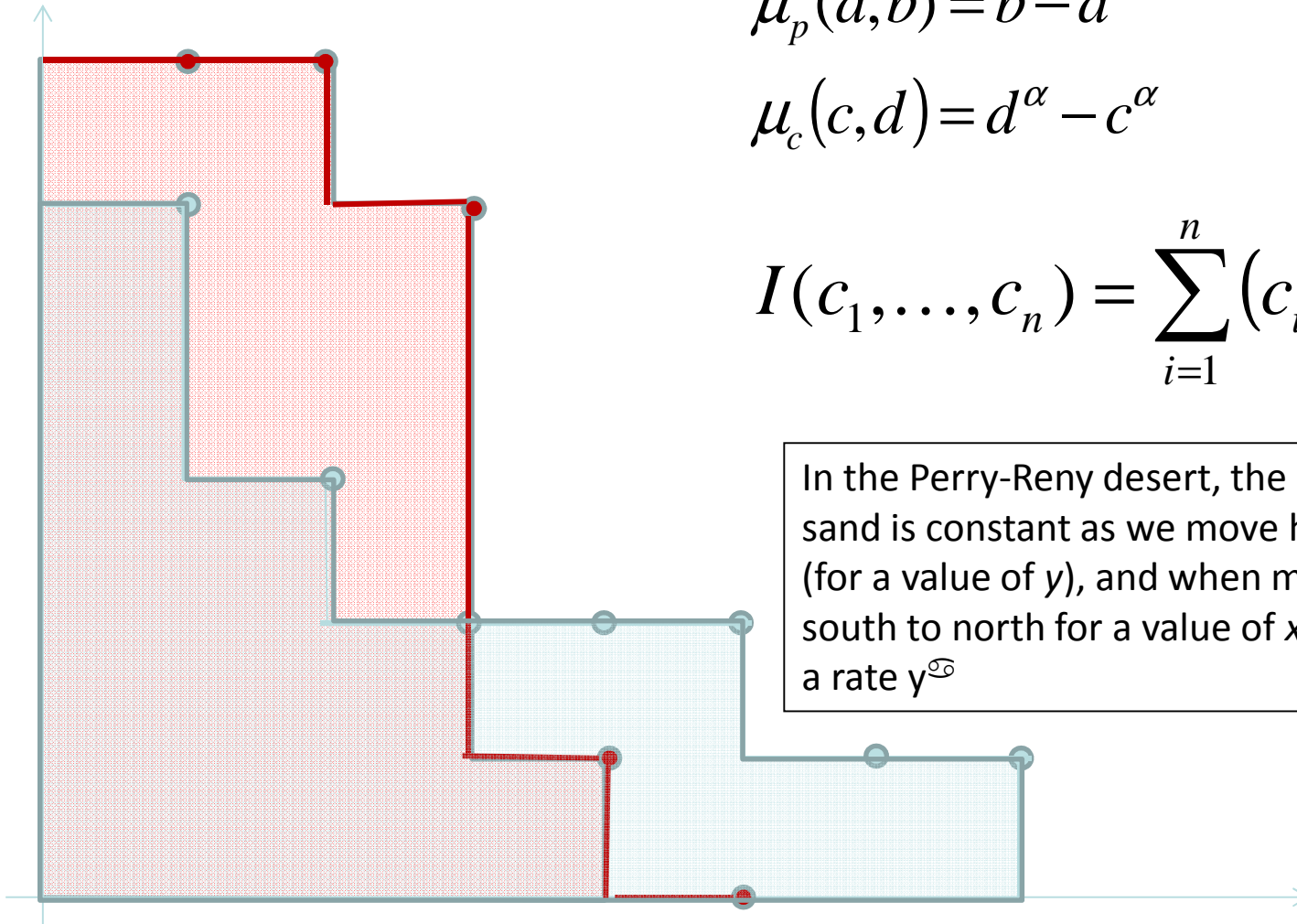
$$\mu[(a,b) \times (c,d)] = \mu_p(a,b) \cdot \mu_c(c,d)$$

$$\mu_p(a,b) = b - a$$

$$\mu_c(c,d) = d^\alpha - c^\alpha$$

$$I(c_1, \dots, c_n) = \sum_{i=1}^n (c_i)^\alpha$$

Citations



In the Perry-Reny desert, the height of the sand is constant as we move horizontally (for a value of y), and when moving from south to north for a value of x it increases at a rate y^α

Publications

Axioms

Monotonicity:

$$(c_1, \dots, c_n) \succeq (c'_1, \dots, c'_n)$$



$$(c_1, \dots, c_n) \succ (c'_1, \dots, c'_n)$$

Axioms

Dummy paper:

$$(c_1, \dots, c_n, 0) \sim (c_1, \dots, c_n)$$

Axioms

Independence:

$$(c_1, \dots, c_n) \succ (c'_1, \dots, c'_n)$$



$$(c_1, \dots, c_n) \circ (x) \succ (c'_1, \dots, c'_n) \circ (x)$$

Axioms

Scale Invariance: for $\lambda > 0$

$$(c_1, \dots, c_n) \succ (c'_1, \dots, c'_n)$$



$$\lambda(c_1, \dots, c_n) \succ \lambda(c'_1, \dots, c'_n)$$

Result (Perry-Reny, 2013)

- An index satisfies **Monotonicity, Dummy paper, Independence** and **Scale invariance** if and only if it is a *monotonic transformation* of

$$\sum_{i=1}^n (c_i)^\alpha$$

Other Developments

- Other theoretical developments (mostly for scientometrics).
- Empirical applications in academic market (Ellison 2012), in the firm (Palacios-Huerta and Prat, 2012).
- Open questions.